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A unified model for inflation and dark energy

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Abstract
We present a model based on the idea that Planck-scale effects may produce a small explicit breaking of global symmetries. The model contains a new complex scalar field Ψ, charged under a certain global $U(1)$ symmetry, interacting with a new real scalar field χ, neutral under $U(1)$. For exponentially small breaking, the model accounts for both *early* and *late* periods of acceleration of the universe.

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1. Introduction

It is well known that symmetries that are broken at a given energy scale may be restored at higher energies. The standard model (SM) of particle physics based on the gauge group $SU(3) \times SU(2) \times U(1)$ describes very well the physics in a wide range of energies, but can only be tested directly up to the highest energy scales reached in particle accelerator experiments. Because of this limitation, we are unable to probe the physics at higher energies in accelerators and see if the SM is still working, or one has to go beyond it. Nevertheless, the universe itself may be regarded as a huge 'laboratory', since when it was just a small fraction of a second old, the energy it contained was enormous, much larger than the maximum energy that can be reached in terrestrial experiments. These first moments of the universe left some imprints on the present observable universe, so that, indirectly, we are able to get some hints about the physics of those huge energies. Some of these hints raise many deep questions for which the SM is unable to give the right answer and this is why many physicists are looking for extensions of the SM. As we are trying to extend the theory from lower to higher energies, it is natural to suppose the existence of additional symmetries, either local, or global. There has been a lot of interest in studying global symmetries at high energies [1–4]. There are reasons to expect that quantum gravity effects break global symmetries: global charges can be absorbed by black holes which may evaporate, 'virtual black holes' may form and evaporate in the presence of a global charge, wormholes may take a global charge away from our universe to another one, etc.

In [2], the loss of quantum coherence in a model of gravity coupled to axions is investigated. The loss of coherence opens the possibility that currents associated with global symmetries are not exactly conserved, while those associated with local symmetries are still exactly conserved. Coleman [3] argued that incoherence is not observed in a many-universe system in an equilibrium state, and pointed out that if wormholes exist they can explain the vanishing of the cosmological constant. The authors of [4] pointed that even if incoherence is not observed in the presence of wormholes, other interesting consequences may emerge, such as the appearance of operators that violate global symmetries, of arbitrary dimensions, induced by baby universe interactions. In this context, the authors of [5] argue that if global symmetries are broken by virtual black holes or topology changing effects, they have to be exponentially suppressed. In particular, in order to save the axion theory, the suppression factor should have an extremely small value $g < 10^{-82}$. This suppression can be obtained in string theory, if the string mass scale is somewhat lower than the Planck scale, $M_{\rm str} \sim 2 \times 10^{18}$ GeV. Thus we expect to have an exponential suppression of the explicit breaking of global symmetries, but, as summarized here, even with such extremely small breaking very interesting cosmological effects may appear [6, 7].

2. The model

In the present contribution we would like to explain our model, which is able to describe both *early* and *late* acceleration periods of the universe. The first one is supposed to have occurred in the very early universe and was given the name *inflation*. The need for an inflationary period of expansion is related to various problems in cosmology, e.g., the flatness and horizon problems, the small-scale inhomogeneities, unwanted relics, etc. The second period of accelerated expansion has started recently, at a redshift $z \sim \mathrm{O}(1)$, and is generically attributed to the so-called *dark energy*. It is suggested by observations of type Ia supernovas [8], the matter power spectrum of large-scale structure [9] and anisotropy of the cosmic microwave background radiation [10].

In our model, the fields responsible for the two accelerating phases are components of a new complex scalar field Ψ that is charged under a certain global $U(1)$ symmetry, with spontaneous breaking scale f. We may write the field Ψ as

$$\Psi = \phi\, \mathrm{e}^{\mathrm{i}\theta/f} \tag{1}$$

and identify the inflaton with the radial part ϕ, and the dark energy field with the angular part θ. We also have a potential containing the following $U(1)$-symmetric term:

$$V_1(\Psi) = \tfrac{1}{4}\lambda(|\Psi|^2 - f^2)^2, \tag{2}$$

where λ is a coupling constant.

In order to satisfy all the constraints to be imposed on any realistic inflationary model, we also introduce a real scalar field χ, neutral under $U(1)$, which interacts with the field Ψ and assists ϕ to inflate. The interaction term is $U(1)$ symmetric and has the form

$$V_2(\Psi, \chi) = \frac{1}{2} m_\chi^2 \chi^2 + \left(\Lambda^2 - \frac{\alpha^2 |\Psi|^2 \chi^2}{4\Lambda^2}\right)^2, \tag{3}$$

with α being a coupling constant and Λ and m_χ are some arbitrary mass scales. Until here, the effective potential only contains $U(1)$-symmetric terms, so that the sum of (2) and (3) represents the symmetric part of the effective potential:

$$V_{\rm sym}(\Psi, \chi) = V_1(\Psi) + V_2(\Psi, \chi). \tag{4}$$

We wish to study the consequences of allowing terms in the potential, which *explicitly* break $U(1)$. Without knowing the details of how Planck-scale physics breaks global symmetries, we introduce the most simple effective $U(1)$-breaking term [11]

$$V_{\text{non-sym}}(\Psi) = -g \frac{1}{M_{\mathrm{P}}^{n-3}} |\Psi|^n (\Psi\, \mathrm{e}^{-\mathrm{i}\delta} + \Psi^\star\, \mathrm{e}^{\mathrm{i}\delta}), \tag{5}$$

where $M_{\mathrm{P}} \simeq 1.22 \times 10^{19}$GeV is the Planck mass and n is an integer satisfying $n > 3$. Because we expect the coupling g to be exponentially suppressed [5], we consider (5) as a small perturbation to the symmetric term V_{sym}. As a consequence, the non-symmetric term can be safely neglected when discussing inflation, but it plays a crucial role at present, being associated with the recent dominating dark energy of the universe.

Thus, the effective potential of our model is given by

$$V_{\text{eff}}(\Psi, \chi) = V_{\text{sym}}(\Psi, \chi) + V_{\text{non-sym}}(\Psi) + C, \tag{6}$$

where C is a constant that sets the minimum of the effective potential to zero.

2.1. *Inflation*

As mentioned above, when discussing inflation in our model, the explicit $U(1)$-breaking term $V_{\text{non-sym}}$ can be neglected and we only take into discussion the symmetric part, namely V_{sym}, of the effective potential. After introducing (1) in the expression for V_{sym}, we obtain the potential only in terms of the fields ϕ and χ:

$$V_{\text{sym}}(\phi, \chi) = \Lambda^4 + \frac{1}{2}\left(m_\chi^2 - \alpha^2\phi^2\right)\chi^2 + \frac{\alpha^4\phi^4\chi^4}{16\Lambda^4} + \frac{1}{4}\lambda(\phi^2 - f^2)^2. \tag{7}$$

This potential is of inverted hybrid inflation type; *hybrid* because the inflaton interacts with the auxiliary field χ whose vacuum energy dominates during inflation, and *inverted* because the inflaton mass term has the changed sign, as compared to typical hybrid inflation models. The mechanism that drives inflation in hybrid models is well described in the literature [12].

There are mainly three constraints that should be imposed on our model in order to fit observations:

- sufficient number of e-folds of inflation $N(\phi) = (8\pi)\big/M_{\mathrm{P}}^2 \int_{\phi_{\text{end}}}^{\phi} (V_{\text{sym}}/V'_{\text{sym}})\, \mathrm{d}\phi$ in order to solve the flatness and the horizon problems. Here, $\phi_{\text{end}} \equiv m_\chi/\alpha$ is the value of the inflaton field at the end of inflation. The number of e-folds that occur after a given scale leaves the horizon is given by [13]

$$N \simeq 62 - \ln\frac{k}{a_0 H_0} - \ln\left(\frac{10^{16}\ \text{GeV}}{\Lambda^{1/4}}\right) + \frac{1}{3}\ln\left(\frac{T_{\text{rh}}}{\Lambda^{1/4}}\right), \tag{8}$$

  where k is the scale that exits the horizon, T_{rh} is the reheating temperature and the biggest explored scale is the present Hubble distance $a_0/k = H_0^{-1} = 6000$ Mpc;
- the amplitude of the primordial curvature power spectrum produced by quantum fluctuations of the inflaton field should fit the observational data [10]

$$\mathcal{P}_{\mathcal{R}}{}^{1/2} = \sqrt{\frac{128\pi}{3}}\, \frac{V_{\text{sym}}(\phi_0, 0)^{3/2}}{M_{\mathrm{P}}^3 V'_{\text{sym}}(\phi_0, 0)} \simeq 4.86 \times 10^{-5}, \tag{9}$$

  where a prime means derivative with respect to ϕ and ϕ_0 is the value of the inflaton field when the scale $k_0 = 0.002\ \text{Mpc}^{-1}$ exits the horizon;

- the value of the spectral index $n_s \simeq 1 - 6\epsilon + 2\eta$ should be in the allowed range suggested by the recent three-year Wilkinson microwave anisotropy probe data [10], $n_s = 0.951^{+0.015}_{-0.019}$. Here $\epsilon \equiv \left(M_P^2/16\pi\right)(V'_{\rm sym}/V_{\rm sym})^2$ and $\eta \equiv \left(M_P^2/8\pi\right)(V''_{\rm sym}/V_{\rm sym})$ are slow-roll parameters. In what follows, we will neglect the ϵ term in the expression of n_s for simplicity, which is a fairly good approximation (less than 0.1% effect on n_s). We obtain

$$\frac{M_P^2}{4\pi}\frac{V''_{\rm sym}(\phi_0, 0)}{V_{\rm sym}(\phi_0, 0)} \simeq -0.05. \tag{10}$$

Combining (8), (9) and (10) we can obtain the dependences of λ, Λ and ϕ_0 on the scale f. The other parameters of our model, namely m_χ and α, do not appear in the previous equations, but there are some relations between them that should be satisfied in order for the hybrid inflation mechanism to work: $\alpha \gg \lambda$, $m_\chi < \alpha f$ and the more important relation

$$f < M_P \tag{11}$$

which sets an upper limit for f. Thus, given the value of f, one is able to calculate the values of the other parameters of our model. Finally, the parameter g does not appear in the discussion about inflation, but will be important when discussing dark energy in our model.

2.2. Dark energy

After inflation, the fields ϕ and χ settle down at the minimum of the symmetric part of the effective potential; the only part 'surviving' after inflation being the non-symmetric term: $V_{\rm non\text{-}sym}(\theta) = -2g(f/M_P)^{n-3}M_P^4\cos(\theta/f)$. The angular field θ can take any value in the range $(0, 2\pi f)$, after the end of inflation. If the potential of θ is sufficiently flat, θ may have a slow rollover up to the present time, such that it may act as a quintessence field and explain the dark energy of the universe. For this to happen, there are two conditions to be satisfied:

- slow-rolling of θ at present, given by the condition

$$m_\theta < 3H_0, \tag{12}$$

  where $m_\theta = \sqrt{2g}(f/M_P)^{(n-1)/2}M_P$ is the mass of θ and $H_0 \sim 10^{-42}$ GeV is the Hubble parameter today (Hubble constant);
- the energy density of θ should be comparable with the present critical density ρ_{c_0}:

$$\rho_\theta \simeq V_{\rm non\text{-}sym}(f, f) \sim \rho_{c_0} \equiv \frac{3H_0^2M_P^2}{8\pi}, \tag{13}$$

  where we supposed that both ϕ and θ are of O(f) today.

Combining (12) and (13) we finally obtain [7]

$$f > \frac{1}{6}M_P \tag{14}$$

$$g < \frac{3\times 6^{n+1}}{8\pi}\frac{H_0^2}{M_P^2}. \tag{15}$$

3. Discussions and conclusions

We have presented a model that can explain *both* inflation and dark energy by introducing a new complex scalar field Ψ with a potential invariant under a new global $U(1)$ symmetry. The radial part of Ψ, namely ϕ, is responsible for inflation when assisted by a new real scalar field, χ, neutral under $U(1)$. The angular field, θ, acquires a tiny mass due to a small explicit breaking of the potential and acts as a quintessence field, explaining the nature of the present dominating dark energy of the universe. The explicit symmetry breaking comes from quantum gravity effects, but we expect them to be exponentially suppressed.

Let us see how much suppression is needed in our model. Equations (14) and (11) indicate that f should be close to the Planck mass. For definiteness, we set it to $f = 0.5M_P$. With $\alpha = 10^{-2}, m_\chi = 3 \times 10^{16}$ GeV and the lowest possible $n = 4$, we obtain for the other parameters of our model the values $\phi_0 = 0.135f, \lambda = 9 \times 10^{-14}, \Lambda = 5.6 \times 10^{15}$ GeV and the limit $g < 10^{-119}$. The constraint on g suggests the level of suppression of the effects of quantum gravity in breaking our $U(1)$ global symmetry. Nevertheless, such tiny values lead to interesting effects for cosmology, e.g., the field θ acts like a quintessence field at present. As a final remark, we should say that this slow-roll regime will not last forever, since when $m_\theta \simeq 3H$, the slow-roll regime will end and θ will start to rapidly oscillate around its minimum.

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